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# Vibration Response of Functionally Graded Material Sandwich Plates with Elliptical Cut-outs and Geometric Imperfections under the Mixed Boundary Conditions

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## Abstract

The present article, investigates the effect of elliptical cut-outs and geometric imperfection on the vibrational response of functionally graded material (FGM) sandwich plates using higher-order shear deformation theory (HSDT). Convergence and validation studies have been performed to demonstrate the efficiency and accuracy of the reported results. The current findings have been achieved using the Lagrangian method based on $C^0$ continuity isoparametric finite element (FE) with four noded elements with seven DOFs per node. The influence of volume fraction index, geometric imperfection modes, and elliptical cut-outs on the vibrational frequency of FGM sandwich plates have been analysed under the mixed boundary constraints.

## Introduction

Functionally graded materials (FGM) are composite materials created by integrating two materials microscopically, and the properties of the material continuously vary from one surface to another.

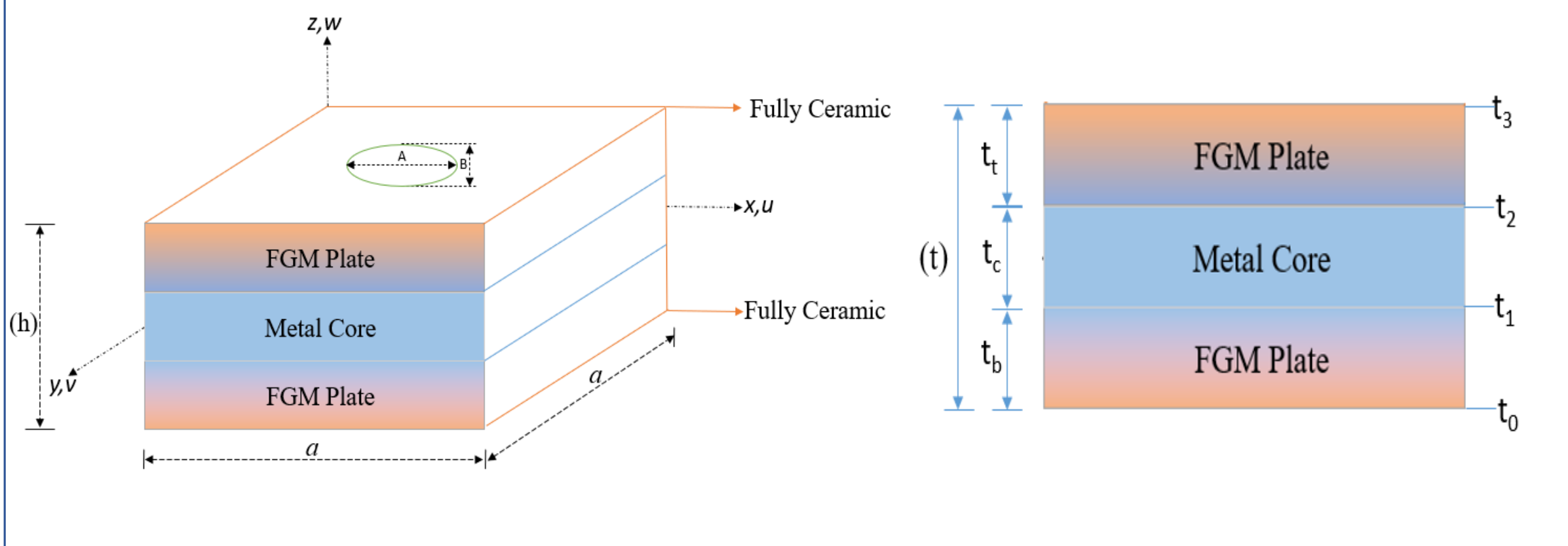

**Figure 1.** Schematic diagram of sandwich FGM plate with elliptical Cut-out

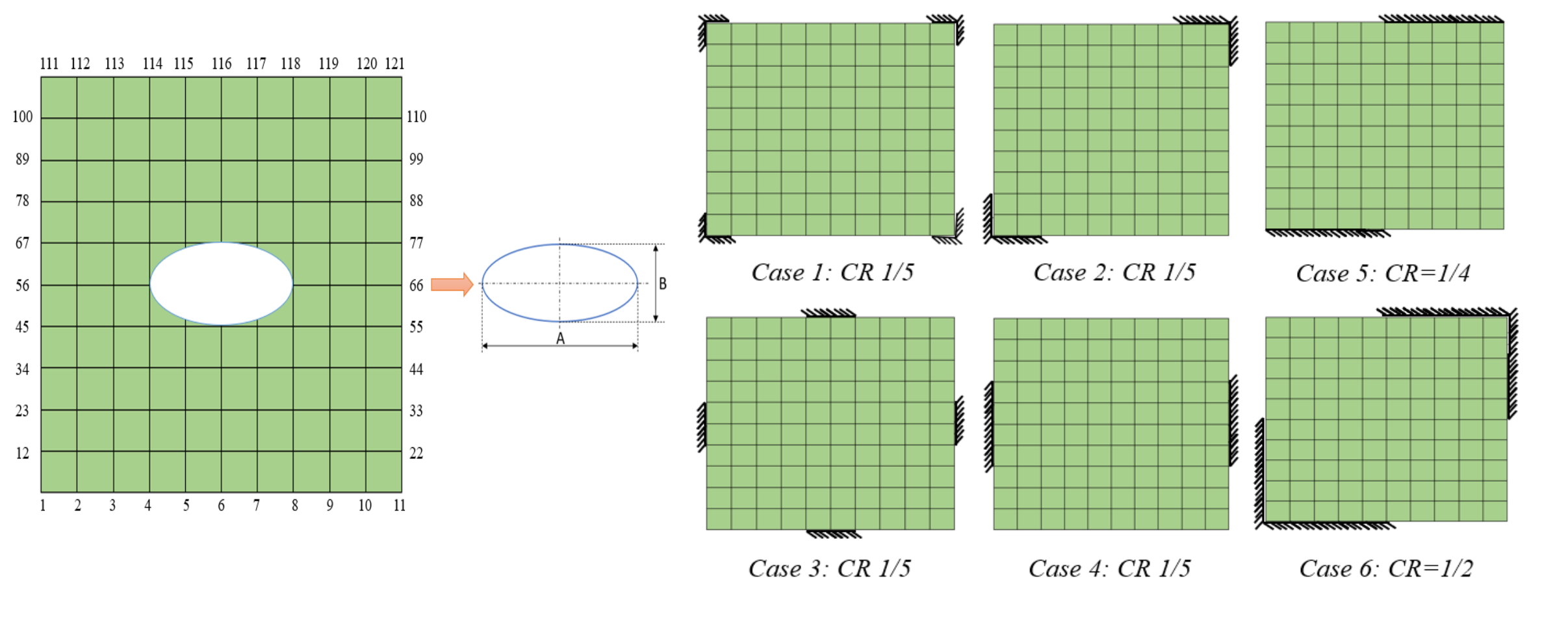

**Figure 2.** Schematic diagram; (i).Plate with Elliptical Cut-out, (ii).Various Mixed BC's

**Table 1.** Material Properties [1] and Non-Dimensional Frequency Parameter [2]

| Material | Properties | | | Non-dimensional frequency parameter |
|---|---|---|---|---|
| | Young's modulus (GPa) | Poisson's Ratio | Density (kg/m³) | $\omega = \bar{\omega}\left(\frac{a^2}{h}\right)\sqrt{\frac{\rho_m}{E_m}}$ |
| Aluminium (Al) (m) | 70 | 0.3 | 2702 | |
| Alumina ($Al_2O_3$) (c) | 380 | 0.3 | 3800 | |

## Mathematical Formulation

- **The displacement field :** The displacement fields are derived using the HSDT, and the trigonometric shear strain function is given by Sarangan and Singh [3], represented as:

$$\begin{aligned} u(x,y,z,t) &= u^0(x,y,t) - \Phi(z)\theta_x - z\varphi_x \\ v(x,y,z,t) &= v^0(x,y,t) - \Phi(z)\theta_y - z\varphi_y \\ w(x,y,z,t) &= w^0(x,y,t) \end{aligned} \qquad (1)$$

Where, $\Phi(z) = \sin\left(\frac{mz}{t}\right)\cos\left(\frac{mz}{t}\right) - \left(\frac{mz}{t}\right)\cos(m)$ the value of 'm' is 2.5.

- **Power law distribution:** The Power law distribution has been used in the present study shown in the Eq. (2) considered from the Literatures.

$$P_e = P_m + (P_c - P_m)V(z) \qquad (2)$$

Where **'P'** represents the effective material property such as subscripts ***c*** and ***m*** denote the **ceramic** and **metal** phases, respectively.

$$\begin{aligned} V^{(b)}(z) &= \left(\frac{z-t_0}{t_1-t_0}\right)^n; z \in [t_0, t_1] \\ V^{(c)}(z) &= 0; z \in [t_1, t_2] \\ V^{(t)}(z) &= \left(\frac{z-t_2}{t_2-t_3}\right)^n; z \in [t_2, t_3] \end{aligned} \qquad (3)$$

Where, ***n*** is the volume fraction of the ceramic phase as given in Eq. 3.

- **The Governing Equation:**

$$[M]\{\ddot{d}\} + [K]\{d_i\} = 0 \qquad (4)$$

- Where, are the [M], [K] global mass and stiffness matrices and $\{\ddot{d}\}$, $\{d_i\}$ are the global acceleration and displacement matrices, respectively.

- **Geometric imperfections:** A wide range of initial imperfection modes can be modelled using this expression given by Kitipornchai et al. [4].

$$Zm = \eta\,\mathrm{sech}\left(\delta_1\left(\frac{X}{a-\psi_1}\right)\right)\cos\left(\pi\mu_1\left(\frac{X}{a-\psi_1}\right)\right)\mathrm{sech}\left(\delta_2\left(\frac{Y}{a-\psi_2}\right)\right)\cos\left(\pi\mu_2\left(\frac{Y}{a-\psi_2}\right)\right) \qquad (5)$$

Where, $\eta$ represents the maximum dimensionless amplitude of the initially deflected geometry $\psi_1$ and $\psi_2$ are the constants defining the localisation degree of the imperfection symmetric about $X_1 = \psi_1$ and $X_2 = \psi_2$ . $\mu_1, \mu_2$ are represents the half-wave numbers of the imperfection in $X_1$ and $X_2$-axis, respectively.

## Results and Discussion

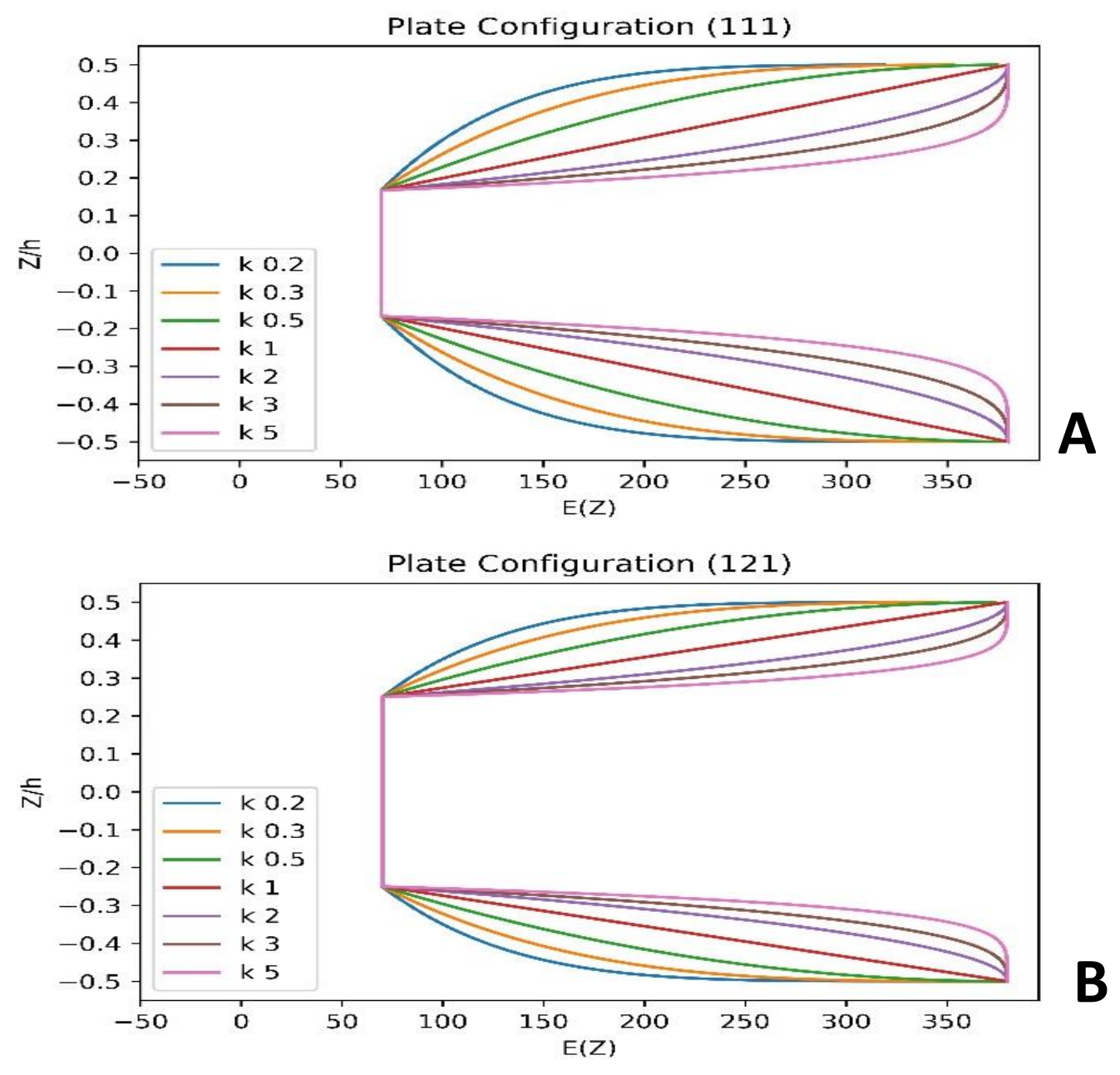

**Figure 3.** Sandwich plate with Configurations, A. (1-1-1), and B. (1-2-1) using $Al/Al_2O_3$ materials

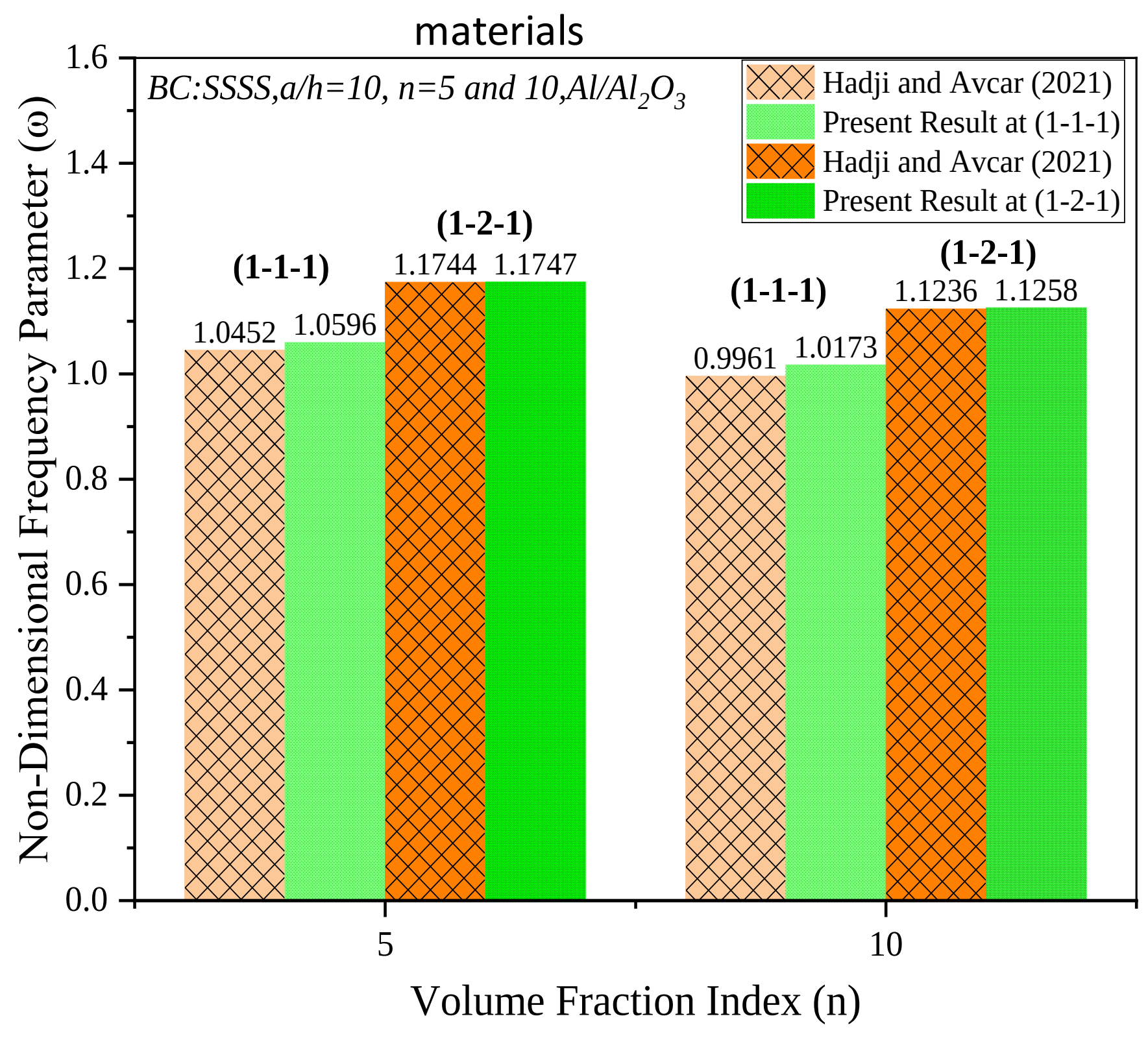

**Figure 4.** Validation results

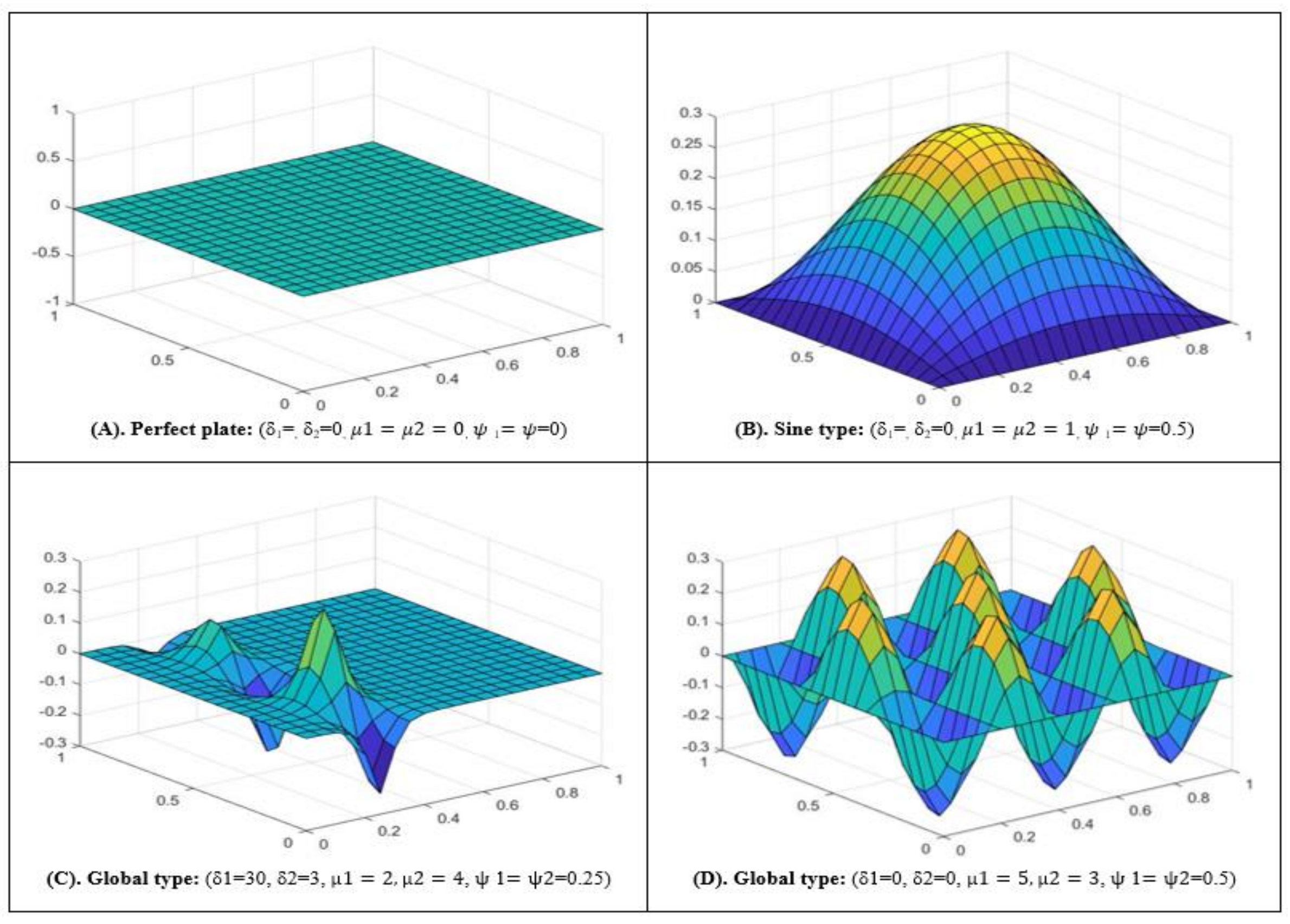

**Figure 5.** Various geometric imperfections [4]

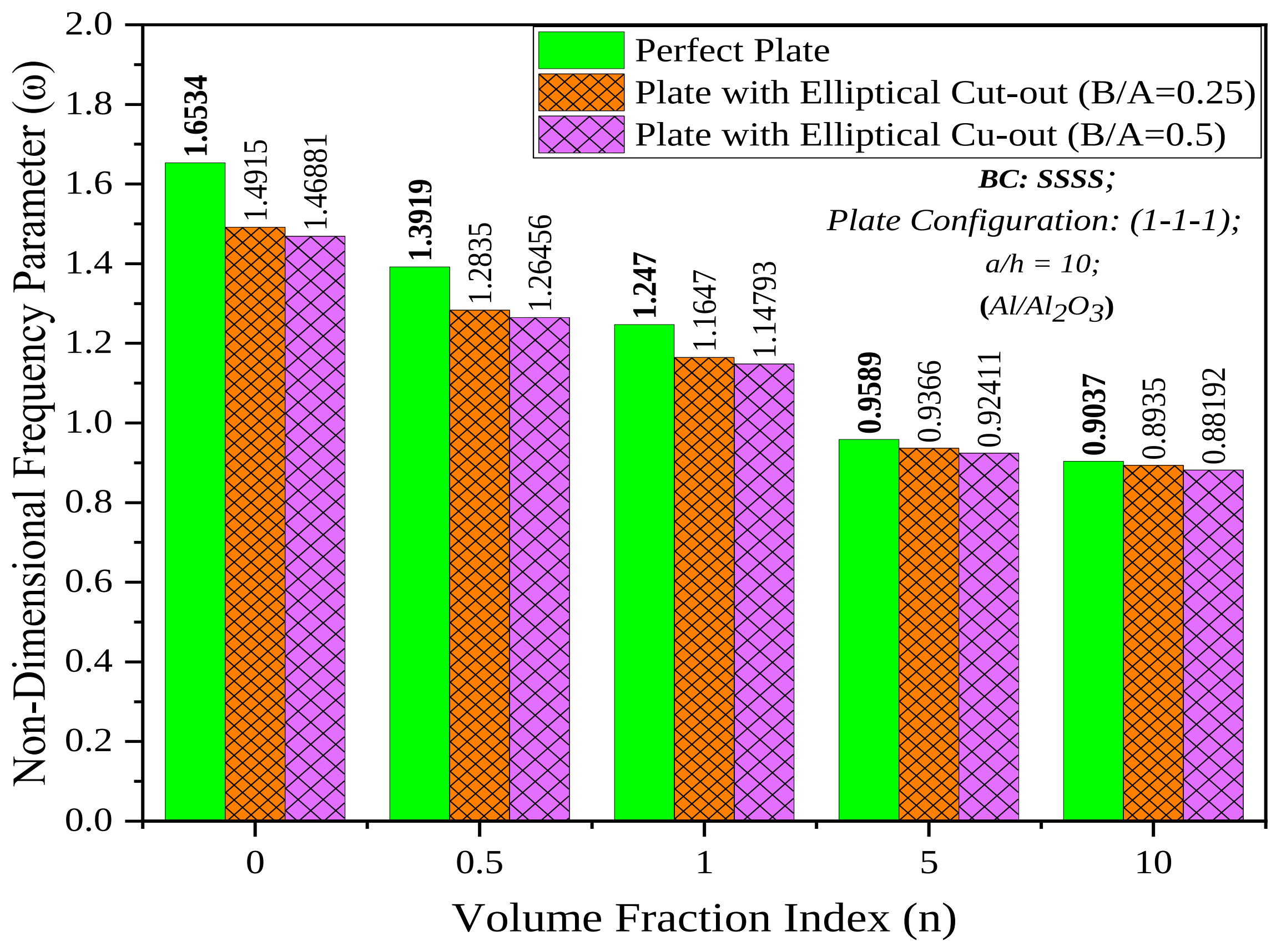

**Figure 7.** Effect of elliptical cut-outs by varying (B/A ratio)

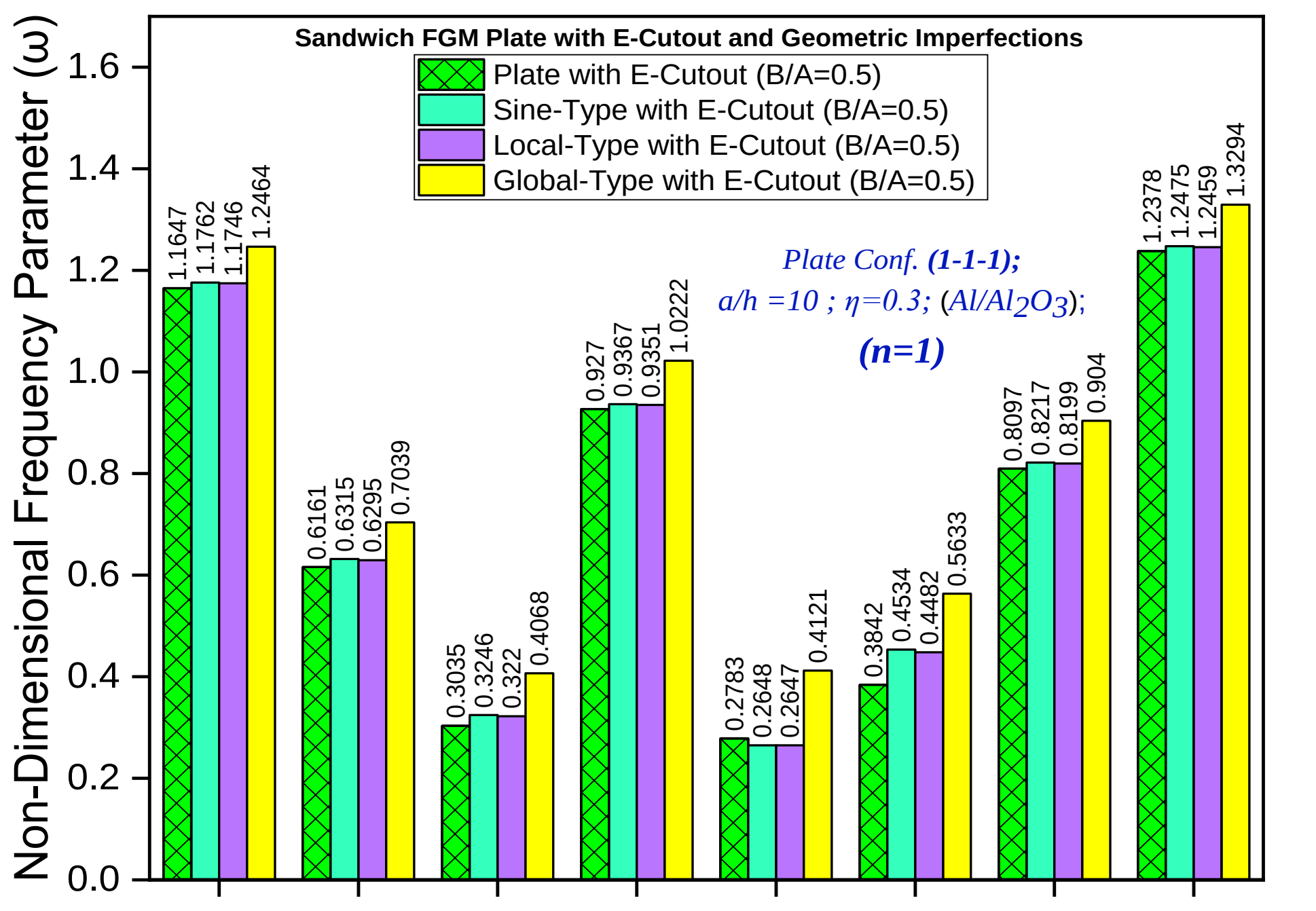

**Figure 9.** Effect of conventional and mixed BC's plate with elliptical cut-out and various imperfections

The validation of NDFP acquired from the present theory with those given by Hadji and Avcar [5] based on HSDT, showing that the results are in excellent agreement as shown in **Fig. 4**

☐ **Influence of geometrical imperfections on the vibration frequency of sandwich FGM plates :**

- It is observed from the results, The influence of imperfection models on the perfect plate at the volume fraction index (n=0) is 0.82% for the sine type, 0.70% for the local type, and 6.28% for the global type.
- Global-type geometric imperfection has a more influence on the frequency parameter than local-type as shown in **Fig. 5 and Fig. 6**

☐ **Influence of elliptical cut-outs on the vibration frequency of Sandwich FGM Plates:**

- The frequency parameter decreases as the elliptical cut-out size increases as shown in **Fig. 7** and **Fig. 8**

☐ **Effect of conventional and mixed boundary conditions on the vibration frequency of Sandwich FGM Plates:**

- Under mixed boundary conditions, Case 3 with (Cr=1/5) has the largest influence frequency parameter and Case 4 with (Cr=1/5) has the lowest influence frequency parameter due to the DOF arrested at different boundaries.

- It's observed that the NDFP increases as the clamping ratio increases from (CR=1/5 to 1/2)shown in **Fig. 9.**

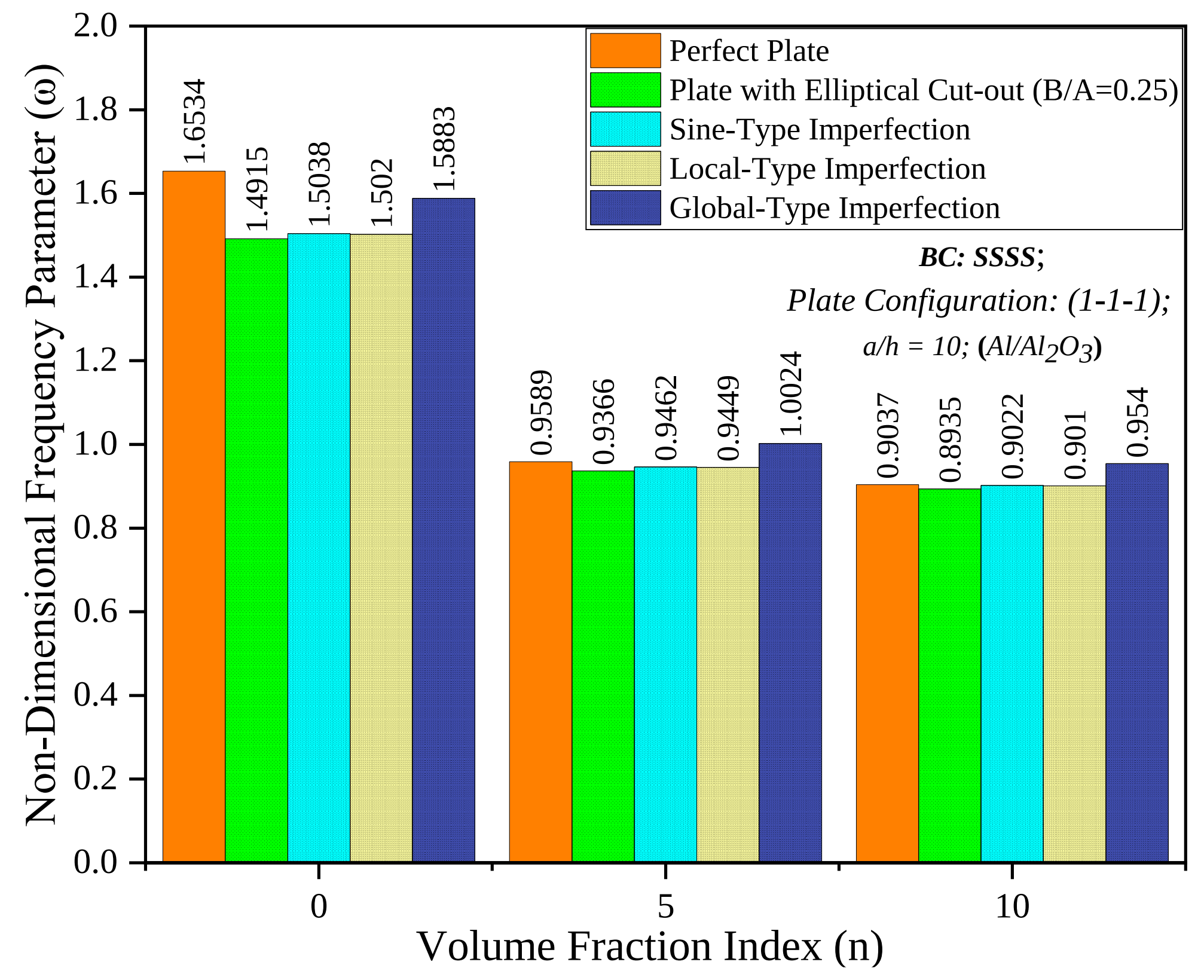

**Figure 6.** Comparison of the NDFP of SFGM square plate with E Cut-out and Geometrically Imperfections

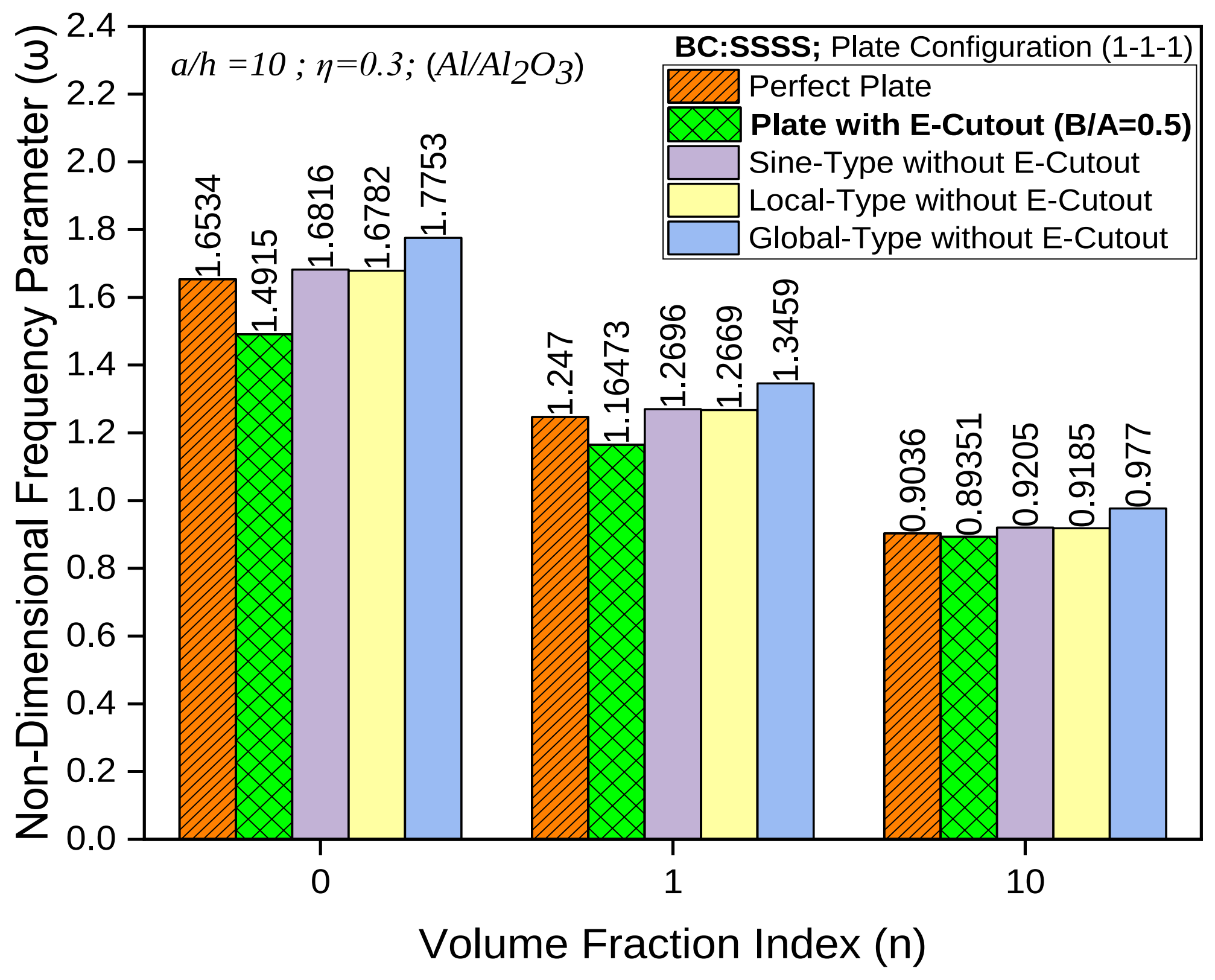

**Figure 8.** Effect of imperfections and Eelliptical -Cut-out

## Conclusions

- The **vibrational frequency** decreases as the **volume fraction index** (*n*) increases because the metallic content increases thereby decreasing the value of stiffness.

- The **vibrational frequencies** of the FGM sandwich plate decrease as the **elliptical-cut-out size** increases.

- It is observed that the **elliptical cut-outs and geometric imperfections** have the **significant influence** on the frequency parameter of the FGM sandwich plates.

- It can be concluded that the vibrational frequency is influenced by the DOF arrested at the boundaries and depends on the location.

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